NAME:_____

**Practice Problems for Borough of Manhattan Community College**
**Math 56 Departmental Final Exam**
**FORM A**

The actual final examination will have 20 questions. Please do not assume that the content or difficulty level of these practice questions is exactly the same as the exam.

Your final exam will be in two parts:

1. **Part 1** of the final does **NOT** allow use of a calculator.
2. **Part 2** of the final **DOES** allow use of a **scientific calculator**. **You may not use a graphing calculator or your phone.**

Note that on this practice exam, the questions are mixed. The symbol  appears next to questions where the use of calculators will be allowed.

**1.** Simplify the following completely and express your answer using only **positive** exponents.

**a.** $\left(\frac{5a^3b^{-2}}{3a^{-5}b^2}\right)^{-3}$ **b.** $\left(\frac{1}{2}\right)^{-4}$ **c.** $(x^2y^{-3})(x^{-5}y^{-1})^5$

**d.** $\frac{(2x^7y^{-1})^2}{(7x^3y)^{-1}}$ **e.** $(-5)^{-3}$

**2.** Perform the indicate operations and simplify completely.

**a.** $(3x - 2)^2 - 3(x-2)(x+2)$

**b.** $x + 3[x - 2(x + 1)]$

**c.** $(3a^2 - 7a + 3) - (2a^2 + 5) + (4a - 1)$

**d.** $(4x-3)(2x^2 - 5x + 7)$

**3.** Factor completely.

**a.** $2x^2 + x - 6$

**b.** $3t^3 - 15t^2 + 18t$

**c.** $25b^2 - 36$

**d.** $2a^3 + 250$

**e.** $2t^3 - 3t^2 - 2t + 3$

**f.** $y^2 - 14y + 48$

**g.** $5a^2 - 45$

**h.** $x^4 - y^4$

**4.** Solve each equation.

**a.** $2(a - 5) - (a - 6) = 5(a - 7)$ (solve for a)

**b.** $\frac{5}{6}x - \frac{3}{4}(6x + 5) + 1 = \frac{2}{3}(9x - 5) + 5$ (solve for x)

**c.** $|4a - 7| + 3 = 6$ (solve for a)

**d.** $2mn - 5xm = 7H$ (solve for x)

**e.** $4x^2 = 81$ (solve for x)

**f.** $x^2 + 7x + 12 = 0$ (solve for x)

**g.** $5(k + 9) - 3 = 2k + 3(k - 1)$ (solve for k)

**h.** $S = pr2 + prs$ (solve for p)

**i.** $|2x + 5| = -2$ (solve for x)

**5.** Solve the following inequalities. Express your solution set on the number line. Write the solution using interval notation.

**a.** $3(2a - 5) - 4a > 10a - 1$

**b.** $|8m - 1| - 2 \geq 4$

**c.** $3|4k + 2| < 9$

**d.** $\frac{3}{4}a - \frac{1}{5} + 4a \leq 6a - \frac{1}{10}$

**e.** $14 < -2x + 6 < 26$

**6.**  The sum of two numbers is 17. One of the numbers is one more than three times the other number. Find the numbers.

**7.**  Doug and Steve went to the candy store. Doug bought 5 pieces of fudge and 3 pieces of bubble gum for $5.70. Steve bought 2 pieces of fudge and 10 pieces of bubble gum for $3.60. How much is one piece of fudge and how much is 1 piece of bubble gum?

**8.** Simona has $10,000 to invest. She invests part of it in an account that yields 2.5%, and the rest in an account that yields 3%. If she earned $283 in interest one year, how much money did she invest in each account?

**9.** The length of a rectangle is 3 less than twice the width. If the rectangle has a perimeter of 30 feet, find the dimensions of the rectangle.

**10.** Graph each line. Use the indicated method.

**a.** $2x - 5y = -10$ (intercept method)

**b.** $3x - 2y + 4 = 0$ (find the slope and y-intercept)

**c.** $y = 5$

**d.** $x = -2$

**11.** Find the equation of each line.

**a.** Find the equation of a line with slope ½ that passes through (-6, 2).

**b.** Find the equation of a line that passes through (-4, 5) and (2, -1).

**c.** Find the equation of a line that passes through (-3, -8) and (-6, -14).

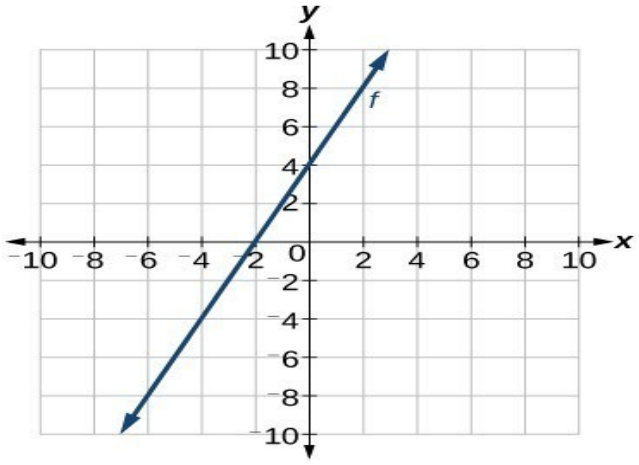
**d.** Find the equation of a line parallel to $3x - 7y = 15$, with an $x$-intercept of 5.

**e.** Find the equation of a line perpendicular to $x - 5y = 10$ and that passes through (-5, -10).

**f.** Find the equation of the line parallel to x = 4 and going through (2,9).

**g.** Find the equation of the line perpendicular to x = 4 and going through (2,9).

**h.** Find the equation of the line on the right.



**12.** Solve each system of linear equations.

**a.** $\begin{cases} -6x - 10y = 20 \\ 3x + 5y = 25 \end{cases}$

**b.** $\begin{cases} 2a - 3b = 6 \\ 5a - 7b = 1 \end{cases}$

**c.** $\begin{cases} y = 5x + 2 \\ 2y - 10x = 4 \end{cases}$

d. $\begin{cases} 3m + 5n = 6 \\ -4m + 2n = 5 \end{cases}$

**13.** Approximate the solution set to the following lines by graphing them. Clearly state the approximate solution.

**a.** $\begin{cases} 2x + 3y = -6 \\ -x + 5y = 10 \end{cases}$

**b.** $\begin{cases} y = -2x + 5 \\ y = 3x - 5 \end{cases}$

**14.** Perform the indicated operations and simplify completely.

**a.** $\frac{x^2-6x+9}{3x-9}$

**b.** $\frac{x^2-2x+1}{3x^2+7x-20} \div \frac{x^2+3x-4}{3x^2-2x-5}$

**c.** $\frac{6c}{3c+15} \times \frac{4c+20}{2c^2}$

**d.** $\frac{2x^2+7x+3}{x^2-16} \div \frac{4x^2+8x+3}{2x^2-5x-12}$

**15.** Divide, find the quotient and remainder. $\frac{7a^3-4a^2-3a+2}{a-4}$

**16.** Find the sum or difference.

**a.** $\frac{x+1}{x-2} - \frac{4x-3}{2x-4} + \frac{x}{x^2-2x}$

**b.** $\frac{3b}{b^2-5b+6} + \frac{b+1}{b^2-b-2}$

**c.** $\frac{2x}{x^2-25} - \frac{1}{5+x}$

**d.** $\frac{5}{3x-5} - \frac{1}{3x}$

**e.** $\frac{x+2}{2x-1} - \frac{4}{2x+1}$

**f.** $\frac{1}{x} - \frac{2x}{x+1}$

**g.** $\dfrac{2}{x^2+5x+6}+\dfrac{x}{x^2-9}$

**17.** Simplify completely and express as a simple fraction.

**a.** $\dfrac{1+\frac{3}{x-2}}{\frac{2x}{x-2}+\frac{4}{x+2}}$

**b.** $\dfrac{2+\frac{3}{x}-\frac{9}{x^2}}{8-\frac{14}{x}+\frac{3}{x^2}}$

**c.** $\dfrac{\frac{2}{3}-6}{\frac{1}{5}-\frac{1}{2}}$

**18.** Solve:

**a.** $\dfrac{3}{8x}-\dfrac{4}{x}=\dfrac{3}{2}+\dfrac{1}{4x}$

**b.** $\dfrac{2}{x-2}+\dfrac{3x}{x^2-4}=\dfrac{4}{x+2}$

**c.** $2-\dfrac{1}{x^2+x}=\dfrac{3}{x+1}$

**d.** $\dfrac{10}{x^2-25}-\dfrac{1}{x-5}=\dfrac{3}{x+5}$

**19.** One number is five times another. The sum of the reciprocals of each number is 3/20. Find the numbers.

**20.** Two cars travel at the same speed. The first car travels 100 miles, while the second car travels 50 miles. If the first car travelled for two more hours than the second car, how many hours did each car travel?

**21.** Simplify the following completely; express all answers using only positive exponents.

**a.** $(27)^{-\frac{2}{3}}$

**b.** $\left(\dfrac{625}{16}\right)^{\frac{3}{4}}$

**c.** $\dfrac{\left(x^{\frac{2}{3}}y^{-3}\right)^{\frac{1}{2}}}{\left(x^{\frac{3}{4}}y^{\frac{1}{2}}\right)^{-1}}$

**d.** $(16y^{-4})^{\frac{3}{2}}$

**22.** Perform the indicated operations; simplify your answer completely:

**a.** $2a^{\frac{1}{2}}\left(3a^{\frac{3}{2}} - a^{\frac{1}{2}}\right)^2$ b. $\dfrac{28x^{\frac{5}{6}} + 14x^{\frac{7}{6}}}{-7x^{\frac{1}{3}}}$

**23.** Simplify the following completely (assume all radicands with variable expressions are non-negative ):

**a.** $\sqrt[3]{16}$ **b.** $\sqrt{80a^3b^4c^9}$ **c.** $\dfrac{3}{\sqrt{5}}$ **d.** $\dfrac{\sqrt{2}}{\sqrt{3}-1}$

**e.** $\dfrac{5\sqrt{3}-2\sqrt{2}}{3\sqrt{2}-1}$ **f.** $\sqrt[4]{48m^{10}n^{15}}$ **g.** $\dfrac{5}{\sqrt[3]{5x}}$

**24.** Find the sum or difference (assume all radicands with variable expressions are non-negative):

**a.** $3x\sqrt{8x} - 4\sqrt{72x^3} + 5x\sqrt{50x}$

**b.** $y\sqrt[3]{8x^4} - 4x\sqrt[3]{27y^3x}$

**25.** Perform the indicated operations, and simplify completely, (assume all radicands with variable expressions represent non-negative numbers).

**a.** $\left(2\sqrt{x} - 3\sqrt{y}\right)^2$ **b.** $\dfrac{\sqrt{a}-b}{\sqrt{a}+b}$

**26.** Solve the following equations:

**a.** $\sqrt[3]{2x-3} + 3 = -1$

**b.** $\sqrt{x-4} + 4 = x$

**c.** $\sqrt{x+7} = x + 5$

**d.** $4 = \sqrt{3a-1}$

**e.** $\sqrt{3x+1} = \sqrt{x-3}$

**f.** $\sqrt{x+3} = \sqrt{x} + 3$

**27.** Perform the indicated operations, simplify your answer completely. Express all answers in standard complex number form.

**a.** (3 – 5i) + (6 + 2i) **b.** (3 – 4i) – (2 – i)

**c.** $(3 - 5i)(6 + 2i)$

**d.** $(4 + 2i)^2$

**e.** $\frac{-4+3i}{-5-2i}$

**28.** Solve each using the indicated method.

**a.** $m^2 - 10m + 8 = 0$ (completing the square)

**b.** $(x - 5)(2x - 1) - 3x = 0$ (quadratic formula)

**c.** $125x^3 + 1 = 0$ (factoring and quadratic formula)

**d.** $2x^2 = -6x - 7$ (use whatever method you want)

**e.** $15x^2 + 11x - 6 = 0$

**f.** $x^2 + 2x - 5 = 0$

**g.** $x^2 + 49 = 0$

**h.** $x^2 + 12 = 0$

**i.** $(x - 3)^2 = 8$

**29.** Solve each logarithmic equation for x.

**a.** $x = \log_{27} \frac{1}{3}$

**b.** $\log 1{,}000 = x$

**c.** $\log_x \frac{1}{64} = 3$

**d.** $\log_6 x = -2$

**e.** $\log_x 8 = \frac{1}{2}$

**f.** $\log_x \frac{1}{2} = -\frac{1}{3}$

**30.** Express the following as a single logarithm:

**a.** $5\log_b x - \frac{1}{4}\log_b y + 2\log_b z$

**b.** $\frac{2}{3}\log_x(a + b) - \log_x a - \log_x b$

**31.** Expand the following using properties of logarithms:

**a.** $\log_5\left(\frac{\sqrt[3]{(x-4)}}{2xy}\right)$

**b.** $\log\left(3x^4\sqrt[4]{y^3}\right)$

**32.** Find the exact value of each.

**a.** $\tan 150^0$

**b.** $\sin(-225^0)$

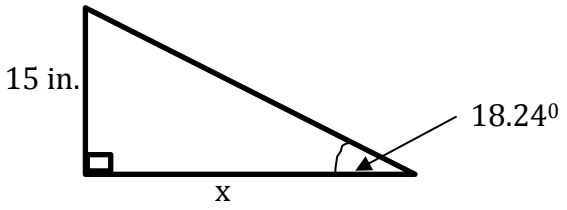
**c.** $\sec 180^0$

**d.** $\cot 390^0$

**33.** If $\sin\propto = \frac{3}{5}$ *and* $\propto$ terminates in Quadrant II, find $\tan \propto$.

**34.** If $\tan\theta = \frac{\sqrt{3}}{4}$, *and* $\theta$ terminates in Quadrant III, find $\csc\theta$.

**35.** Find x. Round your answer to 2 decimal places.

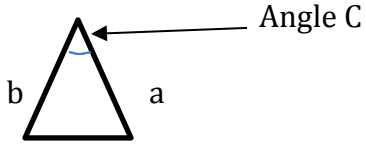



**36.** A 10 foot ladder leans against a building. If the ladder forms an $18^0$ angle with the ground, (a) how high up on the building does the ladder reach? (b) what is the distance from the building to the bottom of the ladder? Round your answers to 2 decimal places.

**37.** A surveyor is standing 50 feet from the base of a tree. If the angle of elevation to the top of the tree is $72.3^0$, how tall is the tree? Round your answer to two decimal places.

**38.** A student stands at ground level, 600 feet away from the base of a building. When she looks up to the top of the building, she finds the angle of elevation is $60^0$. What is the height of the building? Round your answer to two decimal places.

**39.** Find the length of side c in a triangle labeled ABC, if a = 6 cm, b = 10 cm, and $C = 15^0$. Round your answer to 2 decimal places.




**40.** A satellite in outer space is travelling above Houston and Miami. The angle of elevation from Houston to the satellite is $17^0$, and from Miami to the satellite is $21^0$. If the distance from Houston to Miami is 1,200 miles, how far away is the satellite from Houston? Round your answer to 2 decimal places.

**Answer key:**

1.

a) $\frac{27b^{12}}{125a^{24}}$

b) 16,

c) $\frac{1}{x^{23}y^{8}}$

d) $\frac{28x^{17}}{y}$

e) $-\frac{1}{125}$

2.

*a*) $6x^2 - 12x + 16$

b) $-2x - 6$

c) $a^2 - 3a - 3$

d) $8x^3 - 26x^2 + 43x - 21$

3.

a) $(2x - 3)(x + 2)$ *b*) $3t(t - 2)(t - 3)$ *c*) $(5b - 6)(5b + 6)$

d) $2(a + 5)(a^2 - 5a + 25)$ *e*) $(t - 1)(t + 1)(2t - 3)$ *f*) $(y - 6)(y - 8)$

g) $5(a - 3)(a + 3)$ *h*) $(x - y)(x + y)(x^2 + y^2)$

4.

a) $\frac{31}{4}$

b) $x = -\frac{53}{116}$

c) $x = 1$ *or* $x = \frac{5}{2}$

d) $x = -\frac{7H-2mn}{5m}$

e) $x = \pm\frac{9}{2}$

f) $x = -4$ *or* $x = -3$

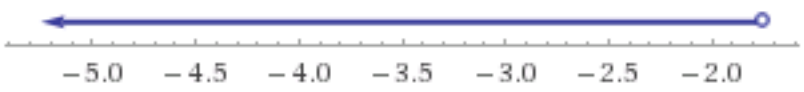
g) *no solution*

h) $p = \frac{s}{r(2+s)}$

i) no solution

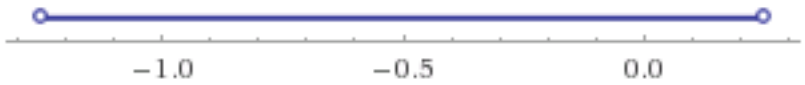
5.

a) a= $(-\infty, -7/4)$

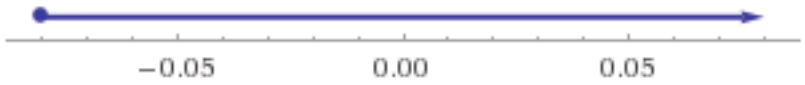


b) $m = \left(-\infty, -\frac{5}{8}\right] \cup \left[\frac{7}{8}, \infty\right)$

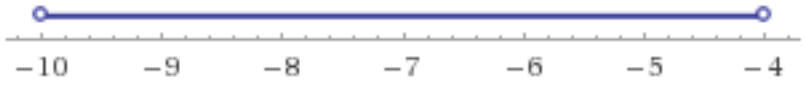

c) $k = \left(-\frac{5}{4}, \frac{1}{4}\right)$




d) $a = \left[-\frac{2}{25}, \infty\right)$




e) $x = (-10, -4)$



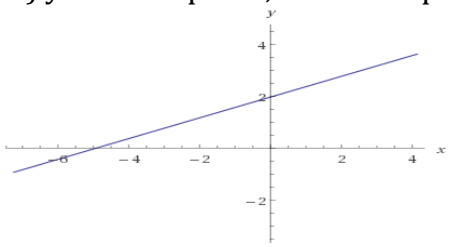

6. 4 ; 13

7. \$1.05/piece of fudge ; \$0.15/piece of bubble gum

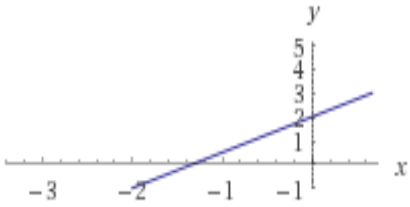
8. \$6,600 ; \$3,400

9. width=6ft ; length=9ft

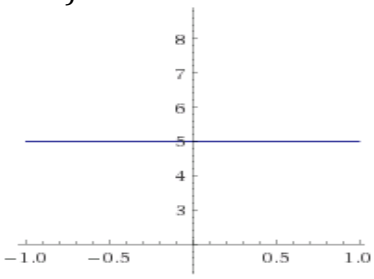
10) a) y – intercept = 2, x – intercept = -5



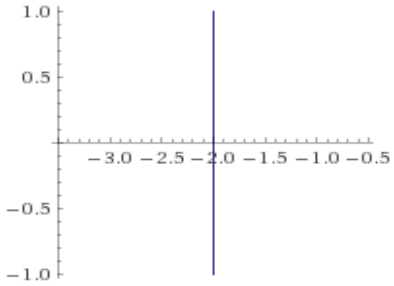

b) slope = 3/2, y-intercept = 2




c)




d)




11.

a) $y = \frac{1}{2}x + 5$

b) $y = -x + 1$

c) $y = 2x - 2$

d) $y = \frac{3}{7}x - \frac{15}{7}$

e) $y = -5x - 35$
f) $x = 2$
g) $y = 9$
h) $y = 2x + 4$

12. a) No solution (lines are parallel); b) (-39,-28); c)Infinitely many solutions – all ordered pairs on the line y= 5x + 2(same line); d) $(-\frac{1}{2}, \frac{3}{2})$

13. a) $(-\frac{60}{13}, \frac{14}{13})$ Quadrant II; b) (2,1) Quadrant I

14. a) $\frac{x-3}{3}$; b) $\frac{(x-1)(x+1)}{(x+4)^2}$; c) $\frac{4}{c}$; d) $\frac{x+3}{x+4}$

15. $7a^2$+24a+93 $+\frac{374}{a-4}$

16. a) $\frac{-2x+7}{2(x-2)}$; b) $\frac{4b-3}{(b-2)(b-3)}$; c) $\frac{1}{x-5}$ ; $d)$ $\frac{12x+5}{3x(3x-5)}$ e) $\frac{2x^2-3x+6}{(2x+1)(2x-1)}$; f) $\frac{-2x^2+x-1}{x(x+1)}$ ; g) $\frac{x^2+4x-6}{(x+3)(x-3)(x+2)}$

17. $\frac{x^2+3x+2}{2x^2+8x-8}$ ; b) $\frac{x+3}{4x-1}$; c) $\frac{160}{9} or\ 17\frac{7}{9}$

18. a) $-\frac{31}{12}$; b) -12; c) $1, -\frac{1}{2}$; d) No Sol. (5 extraneous root)

19. 8; 40
20. 4 hours – first car; 2 hours – second car

21. a) $\frac{1}{9}$ ; b) $\frac{125}{8}$ ; c) $\frac{x^{\frac{13}{12}}}{y}$; d) $\frac{64}{y^6}$
22.
a) $18a^{\frac{7}{2}} - 12a^{\frac{5}{2}} + 2a^{\frac{3}{2}}$
b) $-2(2x^{\frac{1}{2}} + x^{\frac{5}{6}})$

23.
a) $2\sqrt[3]{2}$
b) $4ab^2c^4\sqrt{5ac}$
c) $\frac{3\sqrt{5}}{5}$
d) $\frac{\sqrt{2}(\sqrt{3}+1)}{2}$
e) $\frac{(5\sqrt{3}-2\sqrt{2})(3\sqrt{2}+1)}{17}$
f) $2m^2n^3\sqrt[4]{3m^2n^3}$

g) $\frac{\sqrt[3]{25x^2}}{x}$ , $x \neq 0$

24.
a) $7x\sqrt{2x}$
b) $-10xy\sqrt[3]{x}$

25.
a) $4x - 12\sqrt{xy} + 9y$
b) $\frac{a+b^2-2b\sqrt{a}}{a-b^2}$

26.
a) $x = -\frac{61}{2}$
b) $x = 4 \; or \; x = 5$
c) $x = -3 \; is \; the \; solution, \; ( \; x = -6 \; is \; an \; extraneous \; solution),$
d) $x = \frac{17}{3}$
e) no solution (x = $-2$ is an extraneous solution)
f) no solution (x = 1 is an extraneous solution)
27.
a) $9 - 3i$
b) $1 - 3i$
c) $28 - 24i$
d) $12 + 16i$
e) $\frac{14}{29} - \frac{23i}{29}$

28.
a)$m = 5 \pm \sqrt{17}$
b)$\frac{7\pm\sqrt{39}}{2}$
c) $x = -\frac{1}{5} or \; x = \frac{1\pm i\sqrt{3}}{10}$
d) $x = \frac{-3\pm i\sqrt{5}}{2}$
e) $x = \frac{-11\pm\sqrt{481}}{30}$
f) $x = -1 \pm \sqrt{6}$
g) x = $\pm 7i$
h)x = $\pm 2\sqrt{3}\, i$
i) x = $3 \pm 2\sqrt{2}$

29.
a) $\frac{1}{3}$
b) $x = 3$
c) $x = \frac{1}{4}$

d) $x = \frac{1}{36}$
e) $x = 64$
f) x = 8

30.
a) $\log_b \frac{x^5 z^2}{\sqrt[4]{y}}$

b) $\log_x \frac{\sqrt[3]{(a+b)^2}}{ab}$

31.
a) $\frac{1}{3}\log_5(x-4) - \log_5 2 - \log_5 x - \log_5 y$
b) $\log 3 + 4\log x + \frac{3}{4}\log y$

32.
a) $-\frac{\sqrt{3}}{3}$
b) $\frac{\sqrt{2}}{2}$
c) $-1$
d) $\sqrt{3}$

33. $-\frac{3}{4}$

34. $-\frac{\sqrt{57}}{3}$

35. 45.52ft.

36. a) 3.09ft ; b) 9.51ft.

37. 156.67ft.

38. 1,039.23ft.

39. 4.48cm

40. 698.50mi