

## Diagnostic Practice Questions

**DIAGNOSTIC PRACTICE QUESTIONS** The following questions are organized by topic and include all the possible situations where common errors occur. Check your work. Show all your calculations for each problem in order. Solutions and step-by-step instructions are provided at the end. Indicate which questions you need to “brush up” on. All questions will be addressed during the third hour.

1. Define the following sets of numbers:

a. Counting or Natural Numbers

b. Whole Numbers

c. Integers

d. Rational Numbers

e. Irrational Numbers

f. Real Numbers

2. Indicate which of the following are Real Numbers:

a.  $\sqrt{4}$

b.  $\sqrt{-4}$

c.  $-\sqrt{4}$

3. State which of the following is equal to the Real Number zero or not defined as a Real Number:

a.  $\frac{0}{5}$

b.  $\frac{8}{0}$

c.  $\frac{x}{0}$

d.  $\frac{1}{x}$  ( $x \neq 0$ )

4. State which values of x make the denominator of each of the following fractions equal to zero:

a.  $\frac{1}{x}$

b.  $\frac{1}{3x}$

c.  $\frac{1}{x-2}$

d.  $\frac{1}{x+5}$

e.  $\frac{1}{6x-3}$

f.  $\frac{1}{x^2-9}$

5. Subtract  $-3x^2 + 2x - 8$  from  $-5x^2 + x - 3$ . Simplify completely.

6. Subtract and simplify completely:  $\frac{3}{10} - \frac{3x-4}{10}$

7. Multiply and simplify completely:

a.  $(3x - 5)^2$

b.  $(x - 5)(x + 5)$

c.  $(5 - x)(5 + x)$

d.  $(x - 3)(5x^2 - x - 2)$

8. Simplify completely. Rewrite all expressions without negative exponents.

a.  $5^{-2}$

b.  $(-2)^{-3}$

c.  $(-3)^2$

d.  $-3^2$

e.  $(-2xy^{-4})^3$

f.  $-3x^{-4}$

g.  $(ab)^0$  ( $a \neq 0, b \neq 0$ )

h.  $\frac{x}{x}$  ( $x \neq 0$ )

i.  $\frac{x^3}{x^3}$  ( $x \neq 0$ )

j.  $\frac{2^{-5}}{2^{-2}}$

k.  $\frac{4^{-2}}{4^3}$

l.  $(\frac{x}{y})^2$   $y \neq 0$

m.  $-\frac{3x^4y^7}{6x^5y}$

$x \neq 0, y \neq 0$

n.  $-(-5)^2$

o.  $-18 \div (2)(3) - 1$

p.  $-2(7 - 9)^2 \div 16$

9 Divide:

a.  $\frac{x+8}{x}$   $x \neq 0$

b.  $\frac{x^2+9}{x^2}$   $x \neq 0$

c.  $\frac{x^2-x-12}{x+3}$   $x \neq -3$

d.  $\frac{x-3}{x^2-9}$   $x \neq 3, x \neq -3$

10. Factor the following polynomials completely. If a prime polynomial, state so.

a.  $x^2 - 25$       b.  $36 - x^2$       c.  $3x^3 - 3x$       d.  $14a^2b^3 - 28a^4b^3 + 42a^2b^5$

e.  $x^2 + 9$       f.  $x^2 - 7x - 18$       g.  $2x^2 - 4x - 30$       h.  $2x^2 - 5x - 12$

i.  $4x^3 - 20x^2 - 9x + 45$       j.  $x^2 - 4x + 5$       k.  $x^3 - 2x^2 - 15$

11. Explain the difference between  $x^2 - 3x - 40$  and  $x^2 - 3x - 40 = 0$ .

12. Solve the following equations for x. *If there are no x values that satisfy the equations, state "no solution" or empty set (notation for empty set is:  $\emptyset$ ). If there are infinitely many x values that satisfy the equation, state "infinitely many solutions".*

a.  $5(2x - 3) = 10x - 8$       b.  $2(3x - 4) = 6x - 8$       c.  $8 - (2x - 3) = 5 - 2(5 + 4x)$

d.  $\frac{3x}{4} = -\frac{2}{5}$       e.  $\frac{5x}{4} - \frac{x-2}{12} = -3$       f.  $2x - \frac{1}{3}(2x - 5) = -2 - \frac{x-8}{5}$

13. Rewrite the following equations as equivalent equations in standard form.

Write in descending order of powers and make the leading coefficient positive.

a.  $12 + x - x^2 = 0$       b.  $-x^2 - 5x - 14 = 0$       c.  $30 = x - x^2$       d.  $x(x - 3) = 10$       e.  $2x^2 + 22 = 24x$

f.  $x^2 = 9x$       g.  $100 - x^2 = 0$

14. Solve the following quadratic equations by using the Zero-Factor Property:

a.  $(10x - 5)(x + 3) = 0$       b.  $x(5 - x) = 0$       c.  $x^2 - 81 = 0$       d.  $x^2 - 3x - 28 = 0$

e.  $0 = 7x^2 - 56x$       f.  $x^2 - 6x + 9 = 0$       g.  $3x^2 - 27 = 0$       h.  $12 + x - x^2 = 0$

i.  $x(x - 5) = 24$       j.  $x^2 = 9x$

15. Solve the following equations by using the Zero-Factor Property:

a.  $(x + 1)(x - 3)(5x + 4) = 0$       b.  $5x^3 - 10x^2 - 15x = 0$       c.  $4x^3 - 20x^2 - 9x + 45 = 0$

d.  $x^3 - 4x = 0$

## Solutions to Diagnostic Practice Questions

1a.  $\{1, 2, 3, 4, \dots\}$

1b.  $\{0, 1, 2, 3, 4, \dots\}$

1c.  $\{\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$

1d. Defined as equivalent to  $\frac{a}{b}$ , where  $b \neq 0$ , and  $a$  and  $b$  are integers.

Examples:  $\frac{3}{4} \cdot \frac{1}{2}, 0.45 = \frac{45}{100} = \frac{9}{20}, \sqrt{9} = 3 = \frac{3}{1}$ ,  
*repeating decimal*  $0.333 \dots = \frac{1}{3}, 3.14 = 3\frac{7}{50}$

1e. non-repeating non-ending decimals: Exs:  $\sqrt{2}, \sqrt{3}, \pi = 3.14 \dots \dots$

1f. The Real numbers are comprised of all numbers stated in parts a.-e.. That is, the sets of numbers listed above are subsets of the Real Numbers.

2. a. yes, check:  $(2)^2 = 4$

2b. no, because no real number squared equals  $-4$

2c. yes,  $-1(\sqrt{4}) = -1(2) = -2$  check:  $(-2)^2 = 4$

3 a. yes Real,  $\frac{0}{5} = 0$ , check:  $5 \cdot 0 = 0$

3b. not defined,  $\frac{8}{0} = ?$  check:  $0 \cdot ? \neq 8$

**Note: Division by zero is undefined.**

3c. not defined,  $\frac{x}{0} = ?$  because division by zero is undefined

3d.  $\frac{1}{x}$  is a Real Number when  $x \neq 0$

4a.  $x = 0$

4b.  $x = 0$

4c.  $x = 2$

4d.  $x = -5$

4e.  $x = \frac{3}{6} = \frac{1}{2}$

4f.  $x = 3$  or  $x = -3$

5. Use parentheses to remember to distribute by -1:

$$(-5x^2 + x - 3) - (-3x^2 + 2x - 8) =$$

$$-5x^2 + x - 3 + 3x^2 - 2x + 8 = -2x^2 - x + 5$$

6. Use parentheses to remember to distribute by -1:

$$\frac{3}{10} - \frac{(3x-4)}{10} = \frac{3-(3x-4)}{10} = \frac{3-3x+4}{10} = \frac{7-3x}{10}$$

**Note:  $\frac{10-3x}{10} \neq 1 - 3x$ , Find the common error and write the correct answer.**

7a. **Note:  $(3x - 5)^2 \neq 9x^2 + 25$ . Find the common error and write the correct answer.**

**Note: To multiply the square of a binomial, expand by multiplying the binomial by itself) and then distribute and simplify. Also known as the "F.O.I.L." method.**

$$(3x - 5)(3x - 5) = 9x^2 - 15x - 15x + 25$$

$$= 9x^2 - 30x + 25$$

**Note: This binomial is an example of the "A Perfect Square Trinomial"**

7b.  $(x - 5)(x + 5) = x^2 + 5x - 5x - 25 = x^2 - 25$ .

**Note: This binomial is an example of the "Difference of Two Perfect Squares"**

7c.  $(5 - x)(5 + x) = 25 + 5x - 5x - x^2 = 25 - x^2$

**Note1: This binomial is an example of the "Difference of Two Perfect Squares"**

**Note2: Which of the following is the true statement?**

$$x^2 - 25 = 25 - x^2 \text{ or } x^2 - 25 \neq 25 - x^2$$

7d.  $(x - 3)(5x^2 - x - 2)$

**Note: Distribute six times and then collect like terms.**

$$5x^3 - x^2 - 2x - 15x^2 + 3x + 6 = 5x^3 - 16x^2 + x + 6$$

**Alternate Distribution Method:**

$$(x - 3)(5x^2) + (x - 3)(-x) + (x - 3)(-2) =$$

$$= 5x^3 - 15x^2 - x^2 + 3x - 2x + 6 = 5x^3 - 16x^2 + x + 6$$

8. **Note:**  $(-2)^{-3} \neq 8$  and  $(3)^{-2} \neq -6$  Find the common error and write the correct answer.

**Note: Rules for Exponents**

**Multiply like bases:**  $x^a x^b = x^{a+b}$

**Power to a power:**  $(-cxy^a)^b = (-c)^b x^b y^{ab}$

**Fraction to a power:**  $(\frac{x}{y})^a = \frac{x^a}{y^a}, y \neq 0$

**Dividing like bases:**  $\frac{x^a}{x^b} = x^{a-b}$

**Negative Exponents:**  $x^{-a} = \frac{1}{x^a}$

**Zero Exponent:**  $x^0 = 1$

$$8a. \frac{1}{5^2} = \frac{1}{25}$$

$$8b. \frac{1}{(-2)^3} = -\frac{1}{8} \quad \text{or alternate way:}$$

$$(-2)^{-3} = (-2)^{-1} \cdot (-2)^{-1} \cdot (-2)^{-1} = \frac{1}{-2} \cdot \frac{1}{-2} \cdot \frac{1}{-2} = -\frac{1}{8}$$

$$8c. (-3)^2 = (-3)(-3) = 9$$

$$8d. -3^2 = (-1)(3^2) = (-1)(9) = -9$$

**Note: True or False:**  $(-3)^2 = -3^2$

**Note the use of parentheses in part 8c.**

$$8e. (-2xy^{-4})^3 = (-2)^{1(3)} \cdot x^{1(3)} \cdot y^{-4(3)} = (-2)^3 \cdot$$

$$x^3 \cdot y^{-12} = -\frac{8x^3}{y^{12}}$$

Alternate method:

$$(-2xy^{-4}) \cdot (-2xy^{-4}) \cdot (-2xy^{-4}) = -8 \cdot x^3 \cdot y^{-12} = -\frac{8x^3}{y^{12}}$$

$$8f. -3x^{-4} = (-3)^1 \cdot x^{-4} = -\frac{3}{x^4}$$

$$8g. (ab)^0 = 1$$

$$8h. \frac{x}{x} = x^{1-1} = 1$$

$$8i. x^{3-3} = x^0 = 1$$

**Note:** A polynomial divided by it-self is called a "quantity of one".

State which of the following is true or false:

$$\frac{x+2}{2+x} = 1 \quad \frac{x-3}{-3+x} = 1 \quad \frac{x+5}{x-5} = 1 \quad \frac{x-2}{2-x} = 1$$

$$8j. \frac{2^{-5}}{2^{-2}} = 2^{-5-(-2)} = 2^{-3} = \frac{1}{8}$$

**Alternate method:**

$$\frac{2^{-5}}{2^{-2}} = \frac{2^2}{2^5} = 2^{2-5} = 2^{-3} = \frac{1}{8}$$

$$8k. \frac{4^{-2}}{4^3} = 4^{-2-3} = 4^{-5} = \frac{1}{4^5}$$

$$\text{Alternate method: } 4^{-2} \cdot \frac{1}{4^3} = \frac{1}{4^2} \cdot \frac{1}{4^3} = \frac{1}{4^5}$$

**Note: Simplifying a fraction to a negative power**

$$(\frac{x}{y})^{-2} = \frac{x^{-2}}{y^{-2}} = \frac{y^2}{x^2} \text{ or } (\frac{y}{x})^2$$

**Alternative method**

**Quick method: invert fraction and change negative power to a positive power.**  $(\frac{a}{b})^{-2} = (\frac{b}{a})^2 = \frac{b^2}{a^2}$

$$8l. (\frac{x}{y})^2 = \frac{x^2}{y^2}$$

$$8m. -\frac{3}{6} \cdot \frac{x^4}{x^5} \cdot \frac{y^7}{y} = -\frac{1}{2} \cdot x^{-1} \cdot y^6 = -\frac{y^6}{2x}$$

$$8n. -(-5)(-5) = -25$$

**Note: Order of operations**

**1. Operation(s) within parentheses first**

**2. Exponents next**

**3. then Division and Multiplication-work left to right!!**

**4. then Addition and Subtraction**

$$8o. -\frac{18}{2}(3) - 1 = -9(3) - 1 = -28$$

$$8p. -2(-2)^2 \div 16 = -2(4) \div 16 = -8 \div 16 = -\frac{1}{2}$$

9a. Separate and write as 2 fractions:  $\cancel{\frac{x}{x}} + \frac{8}{x} = 1 + \frac{8}{x}$

**Note:** Find the common error and write the correct

**answer.**  $\cancel{\frac{x+5}{x}} \neq 5$

**Note:** Do not cancel here. "NO PRODUCT, NO CANCEL"

$$9b. \frac{x^2}{x^2} + \frac{9}{x^2} = 1 + \frac{9}{x^2}$$

$$9c. \frac{(x-4)\cancel{(x+3)}}{x+3} = x - 4 \quad \text{factor first to produce a product}$$

$$9d. \frac{\cancel{x-3}}{(x-3)(x+3)} = \frac{1}{x+3}$$

$$10a. (x+5)(x-5) \quad \text{Difference of two perfect squares}$$

$$10b. (6+x)(6-x) \quad \text{Difference of two perfect squares}$$

$$10c. (3x)(x^2 - 1) \quad \text{GCF first}$$

$$= (3x)(x+1)(x-1) \quad \text{Diff. of two squares}$$

$$10d. 14a^2b^3(1 - 2a^2 + 3b^2) \quad \text{GCF}$$

$$10e. x^2 + 9 \quad \text{The sum of two perfect squares is prime!}$$

**Note:** Check to verify that:

$$x^2 + 9 \neq (x+3)(x+3), \quad x^2 + 9 \neq (x-3)(x-3)$$

$$x^2 + 9 \neq (x+3)(x-3)$$

$$10f. (x+2)(x-9) \quad \text{use trial and error method}$$

or use alternate AC method

$$x^2 - 9x + 2x - 18 = x(x-9) + 2(x-9)$$

$$= (x-9)(x+2)$$

**Note:** Always distribute to verify that the signs are

**correct.**  $(x-9)(x+2) = x^2 - 9x + 2x - 18$

$$= x^2 - 7x - 18$$

$$10g. 2(x^2 - 2x - 15) \quad \text{GCF first, then use trial and error method}$$

$$= 2(x-5)(x+3)$$

**Alternative AC method:**

$$\begin{aligned} 2x^2 - 10x + 6x - 30 &= 2x(x-5) + 6(x-5) \\ &= (2x+6)(x-5) = 2(x+3)(x-5) \end{aligned}$$

$$10h. (2x+3)(x-4)$$

$$10i. 4x^2(x-5) - 9(x-5) \quad \text{factor by grouping first- then GCF} = 4x^2 \text{ for first group, and GCF} = -9 \text{ for second group}$$

**REMEMBER TO FACTOR COMPLETELY!**

$$= (x-5)(4x^2 - 9) \quad \text{difference of two squares}$$

$$= (x-5)(2x-3)(2x+3)$$

$$10j. x^2 - 4x + 5 \text{ is a prime polynomial}$$

**Note:** Verify the following by distributing:

$$x^2 - 4x + 5 \neq (x-1)(x-5)$$

$$x^2 - 4x + 5 \neq (x+1)(x+5)$$

$$x^2 - 4x + 5 \neq (x-1)(x+5)$$

$$x^2 - 4x + 5 \neq (x+1)(x-5)$$

$$10k. \text{Prime- a GCF does not exist and not a quadratic expression}$$

**Note:** Quadratic expressions are of the form

$$ax^2 + bx + c$$

$$11. x^2 - 3x - 40 \text{ is quadratic expression}$$

$$x^2 - 3x - 40 = 0 \text{ is a quadratic equation}$$

12a. no solution

$$10x - 15 = 10x - 8 \rightarrow$$

$-15 = -8$  this is a false statement- Since

$-15 \neq -8$  there are no  $x$  values that satisfy the given equation.

$$\frac{12}{1} \cdot \frac{5x}{4} - \frac{12}{1} \cdot \frac{x-2}{12} = \frac{12}{1} \cdot -\frac{3}{1}$$

$$15x - (x - 2) = -36$$

$\rightarrow$  Why is it necessary to use parentheses?

Refer to question #5. Subtracting a polynomial means to distribute the polynomial by  $-1$

$$\rightarrow 15x - x + 2 = -36 \rightarrow 14x = -38 \rightarrow x = -\frac{19}{7}$$

12b. Infinitely many solutions

$$6x - 8 = 6x - 8 \rightarrow -8 = -8 \text{ or}$$

$\rightarrow 0 = 0$  this is a true statement -

Since  $0 = 0$ , there exists infinitely many  $x$  values that satisfy the given equation

Verify by checking (substituting) with any Real Number!

$$12f. x = -\frac{47}{21} \quad \text{L.C.D.} = 15$$

$$\frac{15}{1} \cdot \frac{2x}{1} - \frac{15}{1} \cdot \frac{1}{3}(2x - 5) = \frac{15}{1} \cdot \frac{-2}{1} - \frac{15}{1} \cdot \frac{x - 8}{5}$$

$$\rightarrow 30x - 5(2x - 5) = -30 - 3(x - 8)$$

Why is it necessary to use parentheses?

$$\rightarrow 30x - 10x + 25 = -30 - 3x + 24$$

$$\rightarrow 20x + 25 = -6 - 3x \rightarrow 23x = -31 \rightarrow x = -\frac{31}{23}$$

$$12c. x = -\frac{8}{3}$$

$$8 - 2x + 3 = 5 - 10 - 8x \rightarrow 11 - 2x = -5 - 8x$$

$$\rightarrow 6x = -16 \rightarrow x = -\frac{16}{6} = -\frac{8}{3}$$

$$12d. \frac{3x}{4} = -\frac{2}{5} \quad \text{use cross-multiplication method}$$

$$15x = -8 \rightarrow x = -\frac{8}{15}$$

$$12e. x = -\frac{19}{7}$$

Alternate method to solve a fractional equation

To clear or cancel all denominators, multiply all terms of the equation by the L.C.D.

For this equation, the L.C.D. = 12  $\longrightarrow$

$$13a. x^2 - x - 12 = 0$$

$$13b. x^2 + 5x + 14 = 0$$

$$13c. x^2 - x + 30 = 0$$

$$13d. x^2 - 3x - 10 = 0$$

$$13e. 2x^2 - 24x + 22 = 0$$

$$13f. x^2 - 9x = 0$$

$$13g. x^2 - 100 = 0$$

14. Note: For problem #14, use the Zero-Factor Property

The Zero-Factor Property:

Two factors: If  $A \cdot B = 0$ , then either  $A=0$  or  $B=0$ . ( or both)

Three or more factors: If  $A \cdot B \cdot C \dots \text{etc.} = 0$ , then either  $A = 0$  or  $B = 0$  or  $C = 0$ , etc. (or all factors)

$$14a. 10x - 5 = 0 \text{ or } x + 3 = 0 \rightarrow x = \frac{1}{2} \text{ or } x = -3$$

$$14b. x = 0 \text{ or } 5 - x = 0 \rightarrow x = 0 \text{ or } x = 5$$

**Note: Set the equation equal to zero first and then factor the polynomial.**

$$14c. (x - 9)(x + 9) = 0 \rightarrow x = 9 \text{ or } x = -9$$

$$14d. (x - 7)(x + 4) = 0 \rightarrow x = 7 \text{ or } x = -4$$

$$14e. 7x(x - 8) = 0 \rightarrow x = 0 \text{ or } x = 8$$

$$14f. (x - 3)(x - 3) = 0 \rightarrow x = 3 \text{ or } x = 3$$

**Note for 14f. : These solutions (or roots) are called double roots.**

$$14g. 3(x^2 - 9) = 0 \rightarrow 3(x - 3)(x + 3) = 0$$

$$\rightarrow 3 \neq 0 \text{ or } x = 3 \text{ or } x = -3$$

**Alternate method**

$$\frac{3x^2}{3} - \frac{27}{3} = \frac{0}{3} \rightarrow x^2 - 9 = 0 \rightarrow x = \pm 3$$

$$14h. x^2 - x - 12 = 0 \rightarrow (x - 4)(x + 3) = 0 \rightarrow x = 4 \text{ or } x = -3$$

**14i. Distribute first**

$$x^2 - 5x - 24 = 0 \rightarrow (x - 8)(x + 3) = 0 \rightarrow x = 8 \text{ or } x = -3$$

$$14j. x^2 - 9x = 0 \rightarrow x(x - 9) = 0$$

$$\rightarrow x = 0 \text{ or } x = 9$$

**15a. Note: Since the polynomial is already in factored form, set each factor equal to zero.**

$$x + 1 = 0 \text{ or } x - 3 = 0 \text{ or } 5x + 4 = 0$$

$$\rightarrow x = -1 \text{ or } x = 3 \text{ or } x = -\frac{4}{5}$$

$$15b. 5x(x^2 - 2x - 3) = 0 \rightarrow 5x(x - 3)(x + 1) = 0 \rightarrow 5x = 0 \text{ or } x - 3 = 0 \text{ or } x + 1 = 0$$

$$\rightarrow x = 0 \text{ or } x = 3 \text{ or } x = -1$$

**15c. Factor by grouping**

$$4x^2(x - 5) - 9(x - 5) = 0$$

$$\rightarrow (x - 5)(4x^2 - 9) = 0$$

$$\rightarrow x - 5 = 0 \text{ or } 4x^2 - 9 = 0$$

$$\rightarrow x = 5 \text{ or } (2x - 3)(2x + 3) = 0$$

$$\rightarrow 2x - 3 = 0 \text{ or } 2x + 3 = 0$$

$$\rightarrow x = 5 \text{ or } x = \frac{3}{2} \text{ or } x = -\frac{3}{2}$$

**15d. Factor completely**

$$x(x^2 - 4) = 0 \rightarrow x(x + 2)(x - 2) = 0$$

$$\rightarrow x = 0 \text{ or } x = -2 \text{ or } x = 2$$